Constant Accelerated Life Testing with Geometric Process for Marshall-Olkin Lomax Distribution

Sana Shahab[1], Arif-Ul-Islam[2]

Abstract—We introduce the geometric process for the analysis of accelerated life testing with Marshall-Olkin Lomax distribution for constant stress. By using geometric process one deals with the original parameters of the life distribution in accelerated life testing while in other cases the log linear function between life and stress is used which is a re-parameterization of the original parameter. The maximum likelihood procedure is used for parameter estimation of the model. Simulation-study bootstrapped confidence interval is also evaluated using the R-software. Variation of parameters is also shown.

Index Terms— Geometric Process, Marshall-Olkin Extended Lomax (MOEL) model, Maximum Likelihood Estimation, Fisher Information Matrix, Bootstrapped Confidence Interval, Simulation Study.

1 INTRODUCTION

PRODUCTS with high reliability often require long time period for product life test. In such cases, an Accelerated life test (ALT) is used which is a quick way to obtain information about the life distribution of a material, component or product which reduces the experimental time and the cost incurred in the experiment. It uses aggravated conditions of heat, oxygen, sunlight, vibration, etc. to speed up the normal processes of items. ALT can be used to determine long term effects of expected levels of stress within a shorter time, usually in a laboratory, by controlled standard test methods and estimate the life of a product.

ALT can be carried out using constant-stress, step-stress, or progressive-stress (linearly increasing stress) conditions. In constant stress, each specimen is run at constant stress level while in step-stress loading, a specimen is subjected to successively higher levels of stress. As Compared to step-stress accelerated; constant-stress accelerated life test has some merits such as, simple test methods, ripe theory, and precise test data. In the current study, we have only discussed the application of constant stress in accelerated life testing. Constant stress ALT has been the subject of extensive research, Ahmad et al. [1], Islam and Ahmad [2], Ahmad and Islam [3], Ahmad et al. [4] and Ahmad [5] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring. Yang [6] proposed an optimal design of 4-level constant stress ALT plans considering different censoring times. Pan et al. [7] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by the logistic function. Walkins and John [8] considered the constant stress accelerated life test based on Weibull distribution with constant shape and a log linear link between scale the stress factor which is terminated by a Type-II censoring regime at one of the stress level.

In this paper, the concept of geometric process, first given by Lam [9], is introduced in the context of repair replacement problems. The geometric process simply defines a simple monotone process and has been applied to a variety of situations such as the maintenance problems in engineering. Lam [10] introduced least square and modified moment estimation of parameters for GP, and studied the asymptotic normal properties of these estimators. Lam and Chan [11] derived the maximum likelihood estimate of parameters of the GP with lognormal distribution. Zhou et al. [12] implemented the Geometric Process in the constant stress accelerated life test model based on the progressive Type-I hybrid censored Rayleigh failure data. Sana et al. [17] extended the GP model for the analysis of ALT with complete inverse Weibull failure data under constant stress.

Most of the available literature on accelerated life testing deals with the exponential & weibull distribution. However, these distributions have a limited range of behavior and cannot represent all situations found in applications. Although the exponential distribution is often described as flexible, its hazard function is in fact restricted, being constant .So, a more generalized case is introduced by Marshall and Olkin [13] by adding a new parameter to a family of distribution to overcome the upcoming restrictions. Gupta et al. [14] estimated the reliability from Marshall-Olkin Extended Lomax distribution.

In this paper, the constant stress ALT with geometric process for Marshall-Olkin extended Lomax distribution is taken into consideration. The maximum likelihood estimates of the parameters and bootstrap confidence intervals are calculated through simulation studies. Various graphics are also shown for comparison of various maximum likelihood estimates.

2 GEOMETRIC PROCESS AND MODEL ANALYSIS

2.1 The Geometric Process

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A geometric process describes a stochastic process $\{X_n, n = 1, 2, 3 ....n\}$, where there exists real valued $\lambda>0$ such that $\{\lambda^{n-1} X_n, n = 1, 2, 3 ...n\}$ forms a renewal process. The positive number $\lambda>0$ is called the ratio of GP. It is clear to see that a GP is stochastically increasing if $0<\lambda<1$ and stochastically decreasing if $\lambda>1$. Therefore, the GP is the natural approach to analysed data from a series of event with trend.

2.2 Model Analysis

2.2.1 The Marshall-Olkin distribution

Marshall and Olkin [13] introduced a new method of adding a parameter into a family of distributions. According to, them if $\bar{F}(t)$ denotes the survival or reliability function of continuous random variable X then adding a new parameter results in another Survival function $\bar{G}(t)$ which is defined by

$$\bar{G}(t) = \frac{\delta\bar{F}(t)}{1-\bar{\delta}\bar{F}(t)}, \quad -\infty < t < \infty, \bar{\delta} = 1-\delta \tag{1}$$

If $g(t)$ and $r(t)$ are the probability density function and hazard rate function corresponding to $\bar{G}$ then

$$g(t,\delta) = \frac{\delta\bar{F}(t)}{\left(1-\bar{\delta}\bar{F}(t)\right)^2}, \quad -\infty < t < \infty, \bar{\delta} = 1-\delta \tag{2}$$

and

$$\bar{G}(t) = \frac{h(t)}{1-\bar{\delta}\bar{F}(t)} \tag{3}$$

where $h(t)$ is the hazard rate corresponding to $f(t)$.

Marshall-Olkin extended distributions offer a wide range of behavior than the basic distributions from which they are derived.

2.2.2. Marshall-Olkin extended Lomax distribution

The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of the Marshall–Olkin extended Lomax distribution respectively, are given by

$$g(t;\delta,\alpha,\theta) = \frac{\frac{\delta\alpha}{\theta}\left(1+\frac{t}{\theta}\right)^{\alpha-1}}{\left[\left(1+\frac{t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2} \; ; t>0; \delta,\alpha,\theta>0; \bar{\delta}=1-\delta \tag{4}$$

$$G(t;\delta,\alpha,\theta) = \frac{\left(1+\frac{t}{\theta}\right)^{\alpha}-1}{\left[\left(1+\frac{t}{\theta}\right)^{\alpha}-\bar{\delta}\right]} \; ; t>0; \delta,\alpha,\theta>0; \bar{\delta}=1-\delta \tag{5}$$

α, θ and δ are shape, scale and tilt/index parameter respectively.

2.2.3. Assumptions and test procedure

1. Under any constant stress, the time to failure of a test unit follows a Marshall-Olkin extended Lomax distribution with distribution function given by (5)
2. The shape parameter α is constant.
3. Suppose an accelerated life test with s increasing stress levels in which a random sample of n identical items is placed under each stress level and start to operate at the same time. Let t_{ki}, $k = 1, 2 ...s$, $i = 1, 2, 3 ...n$ denote observed failure time of i^{th} test item under k^{th} stress level.
4. The scale parameter θ_i at stress S_i is given by $\theta_i = e^{a+bS_i}$,where a and b unknown parameters depending on the nature of the product and method of test
5. Let the sequence of random variables $X_0, X_1, X_2, X_4 ....X_n$ denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily. We assume $\{X_k, k = 1, 2, 3 ....s\}$, is a geometric process with ratio $\lambda>0$.

The next proof discusses how the assumption of geometric process (assumption 5) is satisfied when there is a log linear relationship between a life characteristic and the stress level (assumption 4).

From assumption (4), it can easily be shown that

$\log(\theta_k) = a + bS_k$ and $\log(\theta_{k+1}) = a + bS_{k+1}$

It can be written as

$$\log\left(\frac{\theta_{k+1}}{\theta_k}\right) = b\left(S_{k+1}-S_k\right) = b\Delta S = \frac{1}{\lambda} \text{ (Say)} \tag{6}$$

It is clear from (6) that

$$\theta_k = \frac{\theta_{k-1}}{\lambda} = \frac{\theta_{k-2}}{\lambda^2} = ... = \frac{\theta}{\lambda^k}$$

The p.d.f. of product lifetime under k^{th} stress level is

$$g_{X_K}(t) = \frac{\frac{\delta\alpha}{\theta_k}\left(1+\frac{t}{\theta_k}\right)^{\alpha-1}}{\left[\left(1+\frac{t}{\theta_k}\right)^{\alpha}-\bar{\delta}\right]^2} = \frac{\frac{\delta\alpha\lambda^k}{\theta}\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha-1}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2} = \lambda^k \frac{\frac{\delta\alpha}{\theta}\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha-1}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2} \tag{7}$$

Thus from equation (7) we get

$$g_{X_k}(t) = \lambda^k g_{X_0}\left(\lambda^k t\right)$$

Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a geometric process with ratio λ.

ISSN 2226-5518

2.2.4 The random deviate generation

The random deviate can be generated by

$$t=\frac{\theta}{\lambda^k}\left[\left(\frac{u\delta}{1-u}+1\right)^{\frac{1}{\alpha}}-1\right] \qquad 0<u<1 \tag{8}$$

where u ϵ U (0, 1) distribution.

2.2.5 The quantile function

For a continuous distribution $F(t)$ the p percentile ς_p, for a given p, $0<p<1$, is a number such that $P(X\le\varsigma_p)=F(\varsigma_p)=p$.
The quantile function of Marshall-Olkin Extended Lomax (MOEL) model can be obtained by solving:

$$\varsigma_p=\frac{\theta}{\lambda^k}\left[\left(\frac{\delta p}{1-p}\right)^{\frac{1}{\alpha}}-1\right]$$

3 Maximum Likelihood Estimation

While various methods for parameter estimation exist, maximum likelihood estimation (MLE) is one of the most widely used methods. It can be applied to any probability distribution while other methods are somewhat restricted. MLE implementation in ALT is mathematically more complex and, generally, closed form estimates of parameters do not exist. Therefore, numerical techniques such as Newton's method used to compute them.

The likelihood function for constant stress ALT for complete case Marshall-Olkin extended Lomax distribution failure data using GP for s stress levels is given by:

$$L(t;\delta,\alpha,\theta)=\prod_{k=1}^{s}\prod_{i=1}^{n}\lambda^k\frac{\frac{\delta\alpha}{\theta}\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2} \tag{9}$$

The log-likelihood function corresponding (9) can be rewritten as:

$$l=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\begin{array}{l}k\log\lambda+\log\delta+\log\alpha-\log\theta\\+(\alpha-1)\log\left(1+\frac{\lambda^k t}{\theta}\right)-2\log\left(\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-1+\delta\right)\end{array}\right] \tag{10}$$

MLEs of θ, λ and δ are obtained by solving the following normal equations $\frac{\partial l}{\partial\theta}=0,\frac{\partial l}{\partial\lambda}=0$ and $\frac{\partial l}{\partial\delta}=0$ which are given as follows:

$$\frac{\partial l}{\partial\theta}=-\frac{ns}{\theta}-\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{(\alpha-1)}{\theta\left(\frac{\theta}{\lambda^k t}+1\right)}-\frac{2\alpha\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}\left(\frac{\lambda^k t}{\theta^2}\right)}{\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-(1-\delta)}\right] \tag{11}$$

$$\frac{\partial l}{\partial\delta}=\frac{ns}{\delta}-2\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{1}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-(1-\delta)\right]} \tag{12}$$

$$\frac{\partial l}{\partial\lambda}=\frac{ns(s+1)}{2\lambda}+\frac{tk\lambda^{k-1}}{\theta}\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\alpha-1}{\left(\frac{\lambda^k t}{\theta}+1\right)}-\frac{2\alpha\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]}\right] \tag{13}$$

Equations (11), (12) & (13) are used to find the estimate of θ, λ and δ.

4 Fisher Information Matrix & Asymptotic Confidence Interval

The asymptotic Fisher Information matrix is given by:

$$F=\begin{bmatrix}-\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\delta} & -\frac{\partial^2 l}{\partial\theta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\delta\partial\theta} & -\frac{\partial^2 l}{\partial\delta^2} & -\frac{\partial^2 l}{\partial\delta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\theta\partial\lambda} & -\frac{\partial^2 l}{\partial\lambda\partial\delta} & -\frac{\partial l}{\partial\lambda^2}\end{bmatrix}$$

Elements of Fisher Information matrix are:

$$\frac{\partial^2 l}{\partial\lambda^2}=-\frac{ns(s+1)}{2\lambda^2}-\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\begin{array}{l}\frac{2\alpha\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}\left(\frac{tk(k-1)\lambda^{k-2}}{\theta}\right)}{\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-(1-\delta)}\\ -\frac{\frac{t(\alpha-1)}{\theta}k(k-1)\lambda^{k-2}\left(1+\frac{\lambda^k t}{\theta}\right)}{\left(\frac{\lambda^k t}{\theta}+1\right)^2}\\ +\frac{(\alpha-1)\left(\frac{kt\lambda^{k-1}}{\theta}\right)^2}{\left(1+\frac{\lambda^k t}{\theta}\right)^2}\\ -\frac{2\alpha^2\left(1+\frac{\lambda^k t}{\theta}\right)^{2(\alpha-1)}\left(\frac{tk\lambda^{k-1}}{\theta}\right)^2}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2}\\ +\frac{2\alpha(\alpha-1)\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-2}\left(\frac{tk\lambda^{k-1}}{\theta}\right)^2}{\left[\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha}-\bar{\delta}\right]}\end{array}\right] \tag{14}$$

$$\frac{\partial^2 l}{\partial\theta^2}=\frac{ns}{\theta^2}+\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{(\alpha-1)\left(1+\frac{2\theta}{\lambda^k t}\right)}{\left(\frac{\theta^2}{\lambda^k t}+\theta\right)^2}-\frac{2\alpha(\alpha-1)\left(\frac{\lambda^k t}{\theta^2}\right)^2\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-2}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]} -2\alpha\frac{\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}\left(\frac{2\lambda^k t}{\theta^3}\right)}{\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-(1-\delta)}+\frac{2\alpha^2\left(\frac{\lambda^k t}{\theta^2}\right)^2\left(1+\frac{\lambda^k t}{\theta}\right)^{2(\alpha-1)}}{\left[\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2}\right] \tag{15}$$

$$\frac{\partial^2 l}{\partial\delta^2}=-\frac{ns}{\delta^2}+2\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha}-\bar{\delta}\right]^{-2} \tag{16}$$

$$\frac{\partial^2 l}{\partial\lambda\partial\theta}=\frac{\partial^2 l}{\partial\theta\partial\lambda}=\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\theta^2(\alpha-1)\lambda^{-k-1}k}{t\left(\theta+\frac{\theta^2}{\lambda^k t}\right)^2}+\frac{2\alpha\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}\left(\frac{tk\lambda^{k-1}}{\theta^2}\right)}{\left[\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha}-\bar{\delta}\right]} -\frac{2\alpha^2\left(1+\frac{\lambda^k t}{\theta}\right)^{2(\alpha-1)}\left(\frac{tk\lambda^{k-1}}{\theta^2}\right)\left(\frac{\lambda^k t}{\theta^2}\right)}{\left[\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha}-\bar{\delta}\right]^2} +\frac{2\alpha(\alpha-1)\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-2}\left(\frac{tk\lambda^{k-1}}{\theta}\right)\left(\frac{\lambda^k t}{\theta^2}\right)}{\left[\left(1+\frac{t\lambda^k}{\theta}\right)^{\alpha}-\bar{\delta}\right]}\right] \tag{17}$$

$$\frac{\partial^2 l}{\partial\delta\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\delta}=2\alpha\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\left(\frac{tk\lambda^{k-1}}{\theta}\right)\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}}{\left(\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}-\bar{\delta}\right)^2}\right] \tag{18}$$

$$\frac{\partial^2 l}{\partial\delta\partial\theta}=\frac{\partial^2 l}{\partial\theta\partial\delta}=-2\alpha\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\left(\frac{t\lambda^{k}}{\theta^2}\right)\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}}{\left(\left(1+\frac{\lambda^k t}{\theta}\right)^{\alpha-1}-\bar{\delta}\right)^2}\right] \tag{19}$$

Now the variance and covariance matrix can be written as

$$\Sigma=\begin{bmatrix}-\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda} & -\frac{\partial^2 l}{\partial\theta\partial\delta}\\ -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial^2 l}{\partial\lambda^2} & -\frac{\partial^2 l}{\partial\lambda\partial\delta}\\ -\frac{\partial^2 l}{\partial\delta\partial\theta} & -\frac{\partial^2 l}{\partial\delta\partial\lambda} & -\frac{\partial l}{\partial\delta^2}\end{bmatrix}^{-1}$$

$$=\begin{bmatrix}\mathrm{AVar}(\hat\theta) & \mathrm{ACov}(\hat\theta\hat\lambda) & \mathrm{ACov}(\hat\theta\hat\delta)\\ \mathrm{ACov}(\hat\lambda\hat\theta) & \mathrm{AVar}(\hat\lambda) & \mathrm{ACov}(\hat\lambda\hat\delta)\\ \mathrm{ACov}(\hat\delta\hat\theta) & \mathrm{ACov}(\hat\delta\hat\lambda) & \mathrm{AVar}(\hat\delta)\end{bmatrix}$$

The asymptotic confidence interval for α, θ and λ are given by following equations:

$\left[\hat\theta \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat\theta)\right)\right]$, $\left[\hat\lambda \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat\lambda)\right)\right]$ and $\left[\hat\delta \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat\delta)\right)\right]$

5 SIMULATION STUDY

This implementation is done in R. First random samples $u_{ki}(0,1)$, $k = 1, 2, 3, \ldots, s$ $i = 1, 2, 3, \ldots, n$ are generated from a uniform distribution and then with the help of equation (8), t_{ki}, $k = 1, 2, 3, \ldots, s$, $i = 1, 2, 3, \ldots, n$ is generated for $\alpha = 0.9$, $\theta = 0.2$, $\lambda = 0.9$ and $\delta = 0.7$ and the number of stress levels is chosen to be $s = 2$ and 4.

After generating samples for different values of n = 20, 40, 60, 80, 100, 200 & 400, we find the estimate of θ, λ & δ, keeping α (shape parameter) fixed. The function maxNR( ) given in R-package is used for achieving this. It also calculates the functional value, gradient & Hessian.

The performance of the estimates can be evaluated through some measures of accuracy which are mean absolute error (MAE) & square root of mean square error √MSE. Smaller the values of √MSE and MAE better will be the estimated results. Further the 90% bootstrap confidence intervals are also calculated.

5.1 The algorithm of bootstrap confidence Interval

1. First calculate the original sample t_{ki}, $k = 1, 2, 3, \ldots, s$ and $i = 1, 2, 3 \ldots n$ of size n as defined earlier for $s=2$ & $s=4$.
2. For each sample obtain the subsample of size n/2 and obtain MLE
3. Repeat step (2) b=1000 times to get 1000 bootstrap estimates as $\hat\theta^{(1)}, \hat\lambda^{(1)}, \hat\delta^{(1)}$, $\hat\theta^{(2)}, \hat\lambda^{(2)}, \hat\delta^{(2)}$ .... $\hat\theta^{(b)}, \hat\lambda^{(b)}, \hat\delta^{(b)}$.
4. Calculate the average of the estimates
$$\bar\theta=\sum_{j=1}^{1000}\frac{\hat\theta^{j}}{1000}\ ;\ \bar\lambda=\sum_{j=1}^{1000}\frac{\hat\lambda^{j}}{1000}\quad\text{and}\quad\bar\delta=\frac{\hat\delta^{j}}{1000}.$$
5. Now calculate the standard deviation of bootstrap
$$S_\theta=\sqrt{\sum_{j=1}^{1000}\frac{\left(\hat\theta^{(j)}-\bar\theta\right)^2}{1000-1}}$$

Similarly we find other estimates

6. Confidence interval is given by

$\overline{\theta} \pm Z_{\omega} S_{\theta}$; $\overline{\lambda} \pm Z_{\omega} S_{\lambda}$ and $\overline{\delta} \pm Z_{\omega} S_{\delta}$

If 90% confidence interval is required is desired then $Z_{\omega}=Z_{0.05}= 1.645$

TABLE 1

Simulations Results Based on Complete Data from GP Moel with θ=0.2, λ=0.9 and δ=0.7 for s=2

n	Parameter	MLE	SE	MAE	√ MSE	Bootstrap CI LCL	UCL
	θ	3.05730798	1.542300	2.023283	2.248839	0.850262	3.549331
20	λ	1.07504307	0.517166	0.099839	0.126866	0.325226	1.093571
	δ	0.02796739	0.022861	0.210836	0.333843	0.019530	0.046745
	θ	3.84872632	0.970582	2.435263	2.718169	3.592109	4.043534
40	λ	0.78124448	0.266824	0.049884	0.074660	0.468100	0.820670
	δ	0.01428087	0.008603	0.110729	0.042674	0.000129	0.028433
	θ	2.96213065	1.419467	1.909590	2.135449	0.790767	4.956632
60	λ	1.17310157	0.316737	0.111146	0.154519	1.0695420	1.606714
	δ	0.03284991	0.018085	0.210539	0.328151	0.003010	0.076650
	θ	4.84490200	1.007206	2.863792	3.275604	2.175305	5.507042
80	λ	0.98535313	0.226623	0.045073	0.059464	0.612558	1.358148
	δ	0.01847411	0.007042	0.128927	0.235599	0.00689	0.030058
	θ	4.67870811	1.067013	2.652110	3.057382	2.923472	6.433944
100	λ	0.97178306	0.215588	0.039579	0.049930	0.617141	1.326425
	δ	0.01917112	0.007012	0.108742	0.674280	0.018626	0.043060
	θ	3.46291229	0.515048	2.070760	2.347869	2.660587	3.499530
200	λ	0.97805355	0.142969	0.053698	0.065552	0.742860	1.375821
	δ	0.02378003	0.006597	0.128901	0.226727	0.023280	0.034632
	θ	2.75068202	0.213571	1.649567	1.876592	2.3996426	3.824317
400	λ	1.02603535	0.106797	0.076376	0.096369	0.850354	1.296530
	δ	0.03338175	0.005713	0.177863	0.296349	0.0236109	0.047532

TABLE 2

Simulations Results Based on Complete Data from GP Moel with θ=0.2, λ=0.9 and δ=0.7 for s=4

n	Parameter	MLE	SE	MAE	√ MSE	Bootstrap CI LCL	UCL
	θ	4.40133802	1.321414	2.626792	2.986435	1.28739	4.891070
20	λ	1.10482761	0.255569	0.055579	0.086325	0.56470	1.329229
	δ	0.02908435	0.001769	0.140987	0.232817	0.026174	0.031994
	θ	4.08691121	1.439212	2.331112	2.652217	2.790542	5.484916
40	λ	1.01150263	0.106430	0.036467	0.052258	0.836425	1.186589
	δ	0.02655307	0.000114	0.108955	0.197193	0.026219	0.026952

	θ	4.01237101	1.092701	2.04671	1.76890	2.963204	4.015929
60	λ	1.1141263	0.163001	0.01792	0.04892	1.017291	1.118421
	δ	0.02602071	0.000621	0.15290	0.12945	0.022901	0.029102
	θ	4.83951367	0.580893	3.066737	3.471256	3.292285	5.863368
80	λ	1.07103865	0.081181	0.028128	0.058541	0.975301	1.074108
	δ	0.02402091	0.005599	0.246797	0.409150	0.018321	0.044321
	θ	4.23820913	1.424018	2.609324	2.948004	2.063421	5.053179
100	λ	1.04926958	0.069856	0.039496	0.058532	0.474219	1.183426
	δ	0.03011371	0.011980	0.174491	0.283434	0.010487	0.048660
	θ	4.76432543	0.552926	2.894188	3.271238	3.975330	5.021787
200	λ	0.99832867	0.047037	0.027055	0.042626	0.918730	1.290198
	δ	0.02404354	0.001101	0.162330	0.283803	0.022190	0.025855
	θ	4.06432503	0.432106	2.730921	2.841062	3.089364	5.926351
400	λ	1.14927958	0.062971	0.049201	0.030271	0.973662	1.284364
	δ	0.02403981	0.001092	0.183028	0.159429	0.022364	0.029724

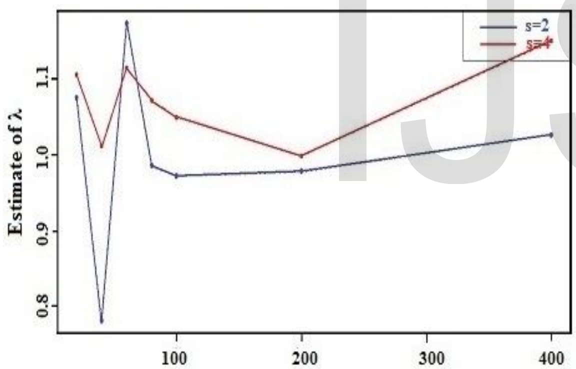

Fig. 1: Plot of estimate of λ of MOEL Model for different stress(s)

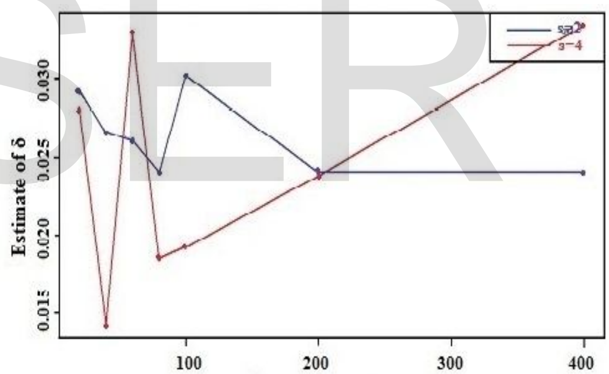

Fig. 2: Plot of estimate of δ of MOEL Model for different stress (s)

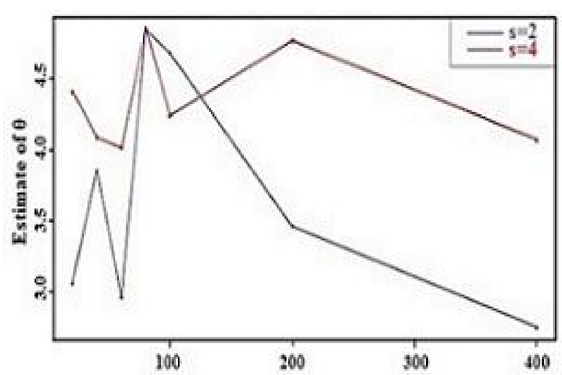

Fig. 3: Plot of estimate of θ of MOEL Model for different stress(s)

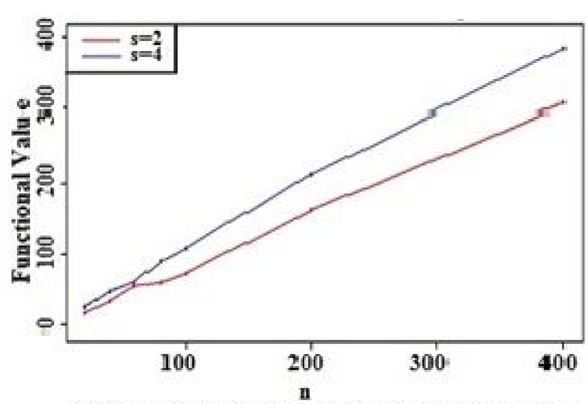

Fig. 4: Plot of Functional Value with change in n

ISSN 2226-5518

6 Conclusion

The proposed geometric method has several innovative and unique features. It deals with original parameter. Table 1 and 2 summarizes the results of the estimates for θ, λ & δ. Based on the results of the simulation study, we observe that estimates $\hat{\theta}, \hat{\lambda}$ and $\hat{\delta}$ are quite well with relatively small root mean squared and mean absolute error. This model work well for stress level $s=4$ and as the value of n increases the estimates become more stable. It should be noted that the bootstrap method demands a further computational complexity, in comparison with approximate confidence interval but it gives better results due to replication. Fig.1, Fig.2 & Fig.3 shows the variation of estimates for different sample values.From Fig. 4, we conclude that how with the increase in sample size the functional value also increases. Hence θ, λ & δ are not sensitive parameters.

Therefore, the test design obtained here is robust design and work well under the situation where no censoring occurs.

Acknowledgment

The first author wishes to thank Mr. Mohd Anjum, Assistant Professor in Aligarh Muslim University for valuable suggestions and help throughout my work which substantially has improved this paper.

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